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# Computers and Renaissance Perspective

Written for Annual Leonardo Meeting, Brescia 1991
Published as Appendix to: *Leonardo's Method*, Brescia: Centro Ricerche Leonardiane, 1993.

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## 1. Introduction

A long tradition of Euclidean geometry developed two-dimensional conventions of representation to the extent that they were part of the legitimation process in mathematics. As a result, Renaissance treatises on perspective evidence a basic paradox: they use abstract two-dimensional conventions to display the principles of a new three-dimensional method of representing space. This is achieved by folding different planes (usually a lateral view and or a ground view) into a single plane (usually a frontal view). This procedure of folding back (technically termed *ribaltimento* in Italian and *rabattement* in French), makes most of the diagrams in the early treatises virtually incomprehensible to the untrained eye, all the more so because the reader is confronted with a completed construction which usually gives no visual clues concerning the steps taken to get there. One can identify the steps taken in arriving at an end product in any of these constructions; one can reconstruct these steps and theoretically it would be possible to print these, except that the cost of including so many diagrams makes this alternative prohibitively expensive. All of which helps explain why these treatises have never been studied systematically.

A project sponsored by the Perspective Institute in the McLuhan Centre at the University of Toronto is exploring some of the new possibilities offered by computer animation. The system presently uses an IBM compatible 286 personal computer with 240 Megabytes and uses AutoCAD 10 for the illustrations. A perspective databank with classification systems, definitions, explanations, bibliography exists on DBASE III Plus and will be integrated with the illustrations using a Geographical Information System (GIS) provided by Generation 5 Cartotechnical Limited. This system will be expanded to include full text, reconstructions and links with digitized paintings from the Canadian Heritage Information Network (CHIN) project and from the Marburg Index, using ICONCLASS. The project is headed by the author; coordinated by Jeff Kemp, with programming by Jerry Szazman, with illustrations and reconstructions by Eric R. Dobbs. This paper uses some of these reconstructions as a starting point for reconsidering the nature of the two major methods of renaissance perspective which Panofsky coined the *legitimate construction* and *distance point construction*. Ironically in trying to present animation techniques in print form the article faces a handicap parallel to that of the Renaissance authors who presented their three-dimensional versions in two-dimensions, with the exception that these limitations in the article have been imposed by technology.

## 2. Window Method

The first treatise on the technical principles of linear perspective was Leon Battista Alberti's *On Painting* (1435). In this work he describes clearly the use of a window (*finestra* or *velo*) and mentions that he does not believe one can make perspectival pictures without using this practical instrument.[1] Hence from the outset perspectival instruments play a central role in perspectival construction. In this same treatise Alberti also gives the first extant description of how to construct a perspectival image. He begins by describing a frontal view: "I find this way to be best. In all things proceed as I have said placing the centric point, drawing the lines from it to the divisions of the base lines of the quadrangle."[2] Alberti then describes the construction of a lateral view:

> In transverse quantities where one recedes behind the other I proceed in this fashion. I take a small space in which I draw a straight line. And this I divide into parts similar to those in which I divided the base line of the quadrangle. Then placing a point at a height equal to the height of the centric point from the base line I draw lines from this point to each division scribed on the first line. Then I establish as I wish the distance from the eye to the picture. Here I draw as the mathematicians say a perpendicular cutting whatever lines it finds. A perpendicular line is a straight line which cutting another straight line makes equal angles all about it. The intersection of this perpendicular line with the others gives one the succession of the transverse quantities.[3]

Alberti does not describe how these two views are correlated. To complicate matters the earliest extant manuscripts of his treatise are without illustrations. This has given rise to three different interpretations. A first claims that parallel lines were simply drawn from the frontal view to the lateral view (fig. 1). This was the interpretation of Klein (1964)[4] and Grayson (1974).[5] A second possibility is that the frontal and lateral views are so aligned that the side of the frontal view is flush with the interposed pole (or window) of the lateral view (fig. 2). This alternative, found in Filarete's *Treatise on Architecture*,[6] is essentially Panofsky's (1927[7] etc.) interpretation. A third possibility is that the frontal and lateral views are so aligned that the central pole of the frontal view is flush with the interposed pole or window (fig. 3) of the lateral view. This alternative, found in the Lucca manuscript[8] of *On Painting*, has led Parronchi (1963)[9] to claim that the legitimate and distance point constructions are equal, as is the case under these special conditions. Using a computer one can reconstruct the frontal and lateral views step by step; align these two views in the same plane and move the interposed pole of the lateral view in order to see how these interpretations are related and to weigh their relative merits. One can also translate the two-dimensional drawing into a three-dimensional situation which reveals that the legitimate construction is simply an abstract version of a practical method using an interposed (surveying) rod or perspectival window (*velo, pariete*, *muro*, fig. 4). Hence the method that Panofsky assumed was an abstract geometrical method began as a practical method, very probably in the context of surveying. A simple extension of this principle can be used to demonstrate how this leads to the perspectival space of a renaissance room (fig. 5).

This method is adopted by Filarete, described by Piero della Francesca in the introduction and then as the second method in book three of his *On the Perspective of Painting* (c.1480), where he relates it for the first time with the combination of ground plan and elevation with which it is now frequently associated. Piero's introductory remarks in this context are worth quoting directly:

> But since in this third [book] I now wish to treat the foreshortenings of objects composed of different surfaces and in different positions [and] because I have to deal with more difficult bodies, I will use another method and way for their foreshortenings which I have not done in the past demonstrations. But in effect it will be the same thing and that which the one does the other does also. But I shall change the past order for two reasons. The one is that it will be easier to demonstrate and to understand. The other is because of the great number of lines that one would have to make following this first method, such that the eye and the intellect would err in these lines and without which lines such bodies cannot be foreshortened in perfection except with great difficulty. Hence I shall adopt this other method.[10]

From the above passage it is clear that Piero is conscious of this method as an alternative which lends itself particularly to objects with complex surfaces. Since books one and two are devoted to the geometrical method and this other method is the subject of only the third book, it would appear that what we now remember as the legitimate construction was not the main method but actually an alternative method during the fifteenth century.

The legitimate construction recurs in Francesco di Giorgio Martini's treatise on geometry[11] where he relates it to surveying experience. Similarly Leonardo da Vinci in his *Manuscript A*[12] begins by showing its basis in surveying experience. Leonardo then proceeds sytematically and demonstrates the principle in terms of a foreshortened square, triangle, diamond-shape, pentagon, hexagon, octagon and a circle. Only then does he draw the foreshortened pavement associated with the legitimate construction adding a diagonal going in the opposite direction beneath which he writes that this is the check (*la ripruova*). If we are right Leonardo is at this point bowing to a tradition that went back to Alberti, whereby the diagonal of the geometrical method served as a legitimation of the practical window method. Yet ironically it is Leonardo's own demonstration that turns the tables and definitively establishes the practical window method as the theoretically based legitimate construction. Hence whereas the geometrical construction had been the prime method until the time of Piero, with Leonardo the legitimate construction emerges as the main method. To confirm the reversibility principle of perspective Leonardo begins with a foreshortened square and works backwards in order to determine the original position of the eye.

These ideas are subsequently disseminated by Dürer (1525), Barbaro (1568) and Danti in his edition of Vignola (1583). The first reference to the alternative method as a legitimate construction is found in Benedetti (1585[13]), which is taken up by Accolti (1625[14]), then revived by Ludwig (1882[15]), and subsequently made famous by Panofsky (1915[16] etc.). Benedetti's testimony is particularly important because he explicitly links the legitimate construction with the window method which he claims is also termed perspective by explanation or unfolding (*prospettiva per dilucidazione*).

## 3. Geometrical Method

A second method of the Renaissance is regularly referred to as the distance point construction,[17] which Panofsky claimed began in the north as a workshop method and did not reach Italy until 1583. This is not true. There are diagrams related to this method in Piero's treatise (Book 1.23) and in Leonardo' notebooks, e.g.on CA173va[18] (c.1490-1495), which suggest that the whole history of this method needs to be reconsidered. Leon Battista Alberti, in his *On Painting* claims that he could give geometrical demonstrations of his principles of perspective if he wished[19] but does not do so. This is just after he describes a procedure in terms of proportion and immediately prior to the description of his *modo optimo* (cf. note 2 above). Our suggestion is that Alberti is implicitly referring to what will emerge as the two main methods of the Renaissance: a principle method which he describes first is based on proportion and becomes the geometrical method of his *Elements of Painting*; then another method which he considers best but describes second because it was less used at the time, which was a prototype of the *legitimate construction* involving the perspectival window or veil (*velo*).

This interpretation helps us to make much more sense of the early history of perspective than provided by the current standard view which claims that Brunelleschi was the great innovator and that it was he who put linear perspective on a sound theoretical footing by linking his demonstration with a ground plan and elevation method. Yet Brunelleschi is described by his biographer Manetti primarily as a practitioner. Maybe so says says the standard view but then he must have been helped by theoreticians such as Paolo dal Pozzo Toscanelli of whose perspective writings nothing definite is (un)fortunately extant. Even if we concede that Brunelleschi did manage this breakthrough (either alone or with help), why then is there no evidence of the ground plan and elevation method in either Alberti or Piero della Francesca? Why is the first extant trace of this method not until c.1480 in Piero della Francesca's treatise, pace Vasari's imaginative claims that it was invented a) by Brunelleschi and b) by Uccello?

Our version of the story is simpler. Brunelleschi made two very dramatic practical demonstrations of linear perspective. As a practitioner he was neither interested or able to write about it. This he left to his friend Alberti who did so in his *On Painting*. In this he referred to two methods: 1) a geometrical method and 2) a combination of frontal and lateral view based on a practical window method. Having only alluded to this first method on his *On Painting* Alberti wrote up the details of this geometrical method in his *Elements of Painting*,[20] which linked the practice of perspective with the traditions of the abaco school and ultimately with Euclid's *Elements*. This was intended to provide a theoretical framework for perspective and helps explain why this first method is alos the first method in Francesco di Giorgio Martini and Piero della Francesca (*On the Perspective of Painting*, books 1 and 2).

If we are right the *legitimate construction* as it is thought of today was unknown to both Brunelleschi and Alberti. The method Alberti described as the best method was his secondary method derived from the practical context of the perspectival window. Moreover This method remained secondary as long as geometrical demonstartion was considered superior to visual evidence. Piero prepared the way for this by introducing the geometrical method with a visual demonstration. Leonardo went a crucial step further by repeating all the fundamental principles of perspective using only the visual method and thus reversing the order of importance of the two methods.

Even if this explanation be not accepted it remains a fact that Alberti's only extensive demonstrations of perspective are found in his *Elements of Painting*, where he describes a geometrical method based on proportion. In this construction the diagonal plays a key role. Since he treats this method in far greater detail there is reason to suggest that it was his major method even though he termed the other method (the later legitimate construction) as the best method. This suggestion is strengthened by the fact noted above, that Piero della Francesca devotes the first two of his three books *On Perspective of Painting* to this geometrical method. With one exception the propositions in these two books depend strictly on geometrical proportions. The exception is book I.23 (fig.6) where Piero begins with a known length and width, i.e. a perspectivally foreshortened rectangle 50 braccia long and 10 braccia wide from which he wishes to cut a square such that it is 10 x 10 braccia. This he proves by reconstructing an unforshortened rectangle and showing their relation in terms of geometrical proportion.[21] He then considers the case of an unknown length and width. From the central vanishing point he draws a line *AO* parallel to the ground, equal in length to the distance of the eye from the interposed plane. Piero then joins the end point *O* of this line with the far side of the interposed plane and produces a diagonal through the foreshortened square. Piero refers to his earlier proposition (I.12) where he appeals to occlusion principles in order to prove that this line from *O* cuts off a square from the rectangle[22]. Hence his proof uses a surveying diagram that serves as the basis for his demonstration of the legitimate construction.

Piero's diagram is actually a combination of at least three different views: a ground plan, frontal view and a lateral view. Using AutoCAD Eric Dobbs has reconstructed the three dimensional situation on which the scene is based . First the diagram is swung into an angular position relative to the screen (fig.7). The ground plane is then folded back (rabatted) 90 degrees until it is directly behind the frontal picture plane (fig.8). The line *AO* which represents the distance of the eye from the picture plane is then swung 90 degrees until it is in front of the picture plane. This is basically the original three dimensional situation which Piero describes (fig.9). If however one now tries to swing the diagonal line by 90 degrees it is found to have its end point 5 feet behind the picture plane rather than 10 (fig.10). Hence this line is not strictly speaking a side view. This problem recurs in the case of the third interpretation of the legitimate construction where the frontal and lateral views are so aligned that the central pole of the frontal view is flush with the interposed pole (or window) of the lateral view (cf. fig.3). If the three dimensional situation is reconstructed, one again finds that the diagonal only reaches to 5 braccia behind the window: i.e half the length of the original tiled square of 10 x 10 braccia (fig. 11).

This detail of the diagonal not being a side view is important for several reasons. It demonstrates how the use of new instruments such as computers can help us to visualize the principles of and also recognize the limitations of earlier methods which we had assumed that we understood. Historically, it suggests that the major method of the fifteenth century originally had nothing to do with a distance point construction; rather that it was a method based on geometrical proportion using diagonals. Originally this diagonal served as a proof that the geometrical proportions were consistent and valid with the result that even in the context of the legitimate construction Alberti and Leonardo continued to refer to this as the check (*pruova* or *ripruova*). As the validity of visual demonstrations gained acceptance, the geometrical method increasingly relied for its proof on the practical window method. Hence, far from being a practical method originating in the workshop practice of the North, it began as an Italian method based on theoretical principles of geometry and proportion. In everyday terms it was handier to use especially in the case of simple planes and objects. However, because this method was not a simple combination of a frontal and lateral view it could not be demonstrated in terms of a folding back (or rabattment) of planes or by a combination of a ground plan and elevation as was the case with the window method. This may well explain why Alberti called the window method the best way (*modo optimo*) and why, after Leonardo had established its foundations it came to be known as the only legitimate construction.

As an engineer, Francesco di Giorgio Martini was concerned with practical demonstrations of perspective. It is notworthy that he too refers specifically to two methods. The first substitutes the principle of thje geometrical method with a surveyor's pole and actual string to trace the paths of the visual rays.[23] Francesco's drawing (fig. 12) is all the more interesting because if the method is applied from both sides one arrives at the so called bifocal method which Uccello had used in his sinopia for the *Nativity* in San Martino alla Scala in Florence (fig. 13). In the course of his practical demonstrations Francesco di Giorgio Martini discovered that it was not necessary to draw all the transverse lines; that a single diagonal could be used for this purpose (fig. 14). This practice was developed by his younger contemporary from Siena, Baldassare Peruzzi, who presented the same idea in more abstract terms (fig. 15) in a method that subsequently became known as the usual rule (*regola ordinaria*).[24] There may have been more behind this name than the particular shortcut introduced by Peruzzi. It may well have been called the *usual rule* because it stemmed from the geometrical method which had been the most common method used by fifteenth and early sixteenth century artists. In any case Francesco di Giorgio's practice based surveying demonstrations of the geometrical method implicitly brought into focus the link between diagonal and distance point in this method, although it appears to have been Jean Pélerin (le Viateur) who first published this in his book *On Artificial Perspective* (1505).[25]

Francesco di Giorgio's second method is effectively Alberti's window method, except that he uses a surveyor's pole instead of a window for the purpose of the intersections and limits himself to showing only the lateral view.

**4. Conclusions**

Ever since Panofsky it has been assumed that the Renaissance had two chief methods one the *legitimate construction* which was theoretical and Italian; the other a *distance point construction* which was derived from workshop practice in the North which only reached Italy in the late sixteenth century. We have shown that major authors of the fifteenth century (implicity Alberti and Leonardo da Vinci, explicitly Piero della Francesca and Francesco di Giorgio Martini) were familiar with two methods; that both these methods had Italian origins; that what Panofsky termed the legitimate construction actually originated as a practical method using a perspectival window; while the method that he termed a distance point construction began as a theoretical geometrical method; that by the early sixteenth century both the practice based legitimate construction and theory based geometrical method were seen to have concrete equivalents, yet were typically expressed in terms of abstract geometrical diagrams which used folding back of planes (*rabattment*) to align various views in a single plane. As the sixteenth century progressed the practical side of these two methods was increasingly forgotten and the diagrams were treated as if they were indeed abstract demonstrations. Gradually what had been a practical window method became remembered as a theoretical legitimate construction and what had begun as a theoretical geometrical construction became associated with a practical distance point construction.

Some will rightly see in this story a vivid example of how significant have been schools such as the neo-Kantians in changing interpretations of texts. Others will find this a striking illustration of shifting criteria for proof. At the outset of the Renaissance it was assumed that Euclid's *Elements* were the equivalent of an ultimate authority with respect to geometry and all related fields. As Aiken[26] has rightly noted, this heritage was so strong that the new possibilities of three-dimensional spatial representation introduced by the discovery linear perspective were faithfully recorded and published in the outmoded two dimensional conventions of the Greek geometrical tradition. There was no sudden revolution partly because the details were kept secret, partly because humanists such as Alberti or architects such as Filarete did not rely on spatial diagrams to illustrate their principles. It took nearly three quarters of a century before the geometrical proofs of Alberti led to the visual proofs of Piero della Francesca and Leonardo da Vinci and before the spatial implications of perspective began to emerge, and persons recognized that both perspective and optics could not hope to be precise without the help of instruments.[27]

In this context the use of computers to elucidate principles of Renaissance perspective takes on a special significance. For the extension of criteria for optical veridity from the unaided eye to include both instruments of vision and representation marked an important step in relating individual experience to a framework that is common to many and therefore more objective. The computer essentially takes this process of objectification one step further and thus marks a further implication of the principles of linear perspective. Perhaps that it why perspective is witnessing a renaissance of interest. Computers are finally helping us to see more clearly the monumentality of the shift that perspective brought to the Renaissance: not a new way of seeing but a new tool for recording and verifying our many ways of seeing.

**Notes**

[1] Leon Battista Alberti, *De pictura*, ed. Cecil Grayson, Rome, Bari: Laterza, 1975, p.56: *Non credo io dal pittore si richiegga infinita fatica, ma bene s'saspetti pittura quale molto molto paia rilevata e simigliata a chi ella si ritrae; qual cosa non intendo io sanza aiuto del velo alcuno mai possa.*

[2] Ibid p.38: *Trovai adunque io questo modo ottimo cosi in tutte le cose seguendo quanto dissi, ponendo il punto centrico, traendo indi linee alle divisioni della giacente linea del quadrangolo.*

[3] Ibid: *Ma nelle quantità trasverse, come l'una seguiti l'altra cosi cognosco. Prendo uno picciolo spazio nel quale scrivo una diritta linea, e questa divido in simile parte in quale divisi la linea che giace nel quadrangolo. Poi pongo di sopra uno punto alto da questa linea quanto nel quadrangolo posi el punto centrico alto dalla linea che giace nel quadrangolo e da questo punto tiro linee a ciascuna divisione segnata in quella prima linea. Poi costituisco quanto io voglia distanza dall'occhio alla pittura, e ivi segno, quanto dicono i matematici, una perpendiculare linea tagliando qualunque truovi linea. Dicesi linea perpendiculare quella linea dritta, quale tagliando un'altra linea diritta fa appresso di sé di qua e di qua angoli retti. Questa cosi perpendiculare linea dove dall'altra sarà tagliata, cosi mi darà la sucessione di tutte le trasverse quantità. E a questo modo mi truovo descritto tutti e' paraleli, cioe le braccia quadrate del pavimento nella dipintura.*

[4] Robert Klein, "Pomponius Gauricus on perspective", *Art Bulletin*, New York, XLIII, 3, sept. 1961, 211-230.

[5] . Cecil Grayson, "L. B. Alberti's 'costruzione legittima'", *Italian Studies*, XIX, 1964, pp.14-27.

[6] Antonio Averlino (il Filarete), *Trattato di architettura*, ed. Anna Maria Finoli e Liliana Grassi, Milan: Edizioni il Polifilo, 1972, vol.1, Tav.129-130 (f.174rv).

[7] Erwin Panofsky, "Die Perspektive als symbolische Form", *Vorträge der Bibliothek Warburg 1924-1925*, Leipzig, 1927, pp. 258-330.

[8] Gino Arrighi, "Il modo optimo dell'Alberti per la costruzione prospettica", *Physis*, Florence, XIV, n.3, pp.295-298

[9] Alessandro Parronchi, "La 'costruzionne legittima' e uguale alla 'costruzione con punti di distanza'", *Rinascimento*, Florence, ser. 2, vol. 3, 1963, 34-40.

[10] Piero della Francesca, *De prospectiva pingendi*, ed. G. Nicco Fasola, Florence:Sansoni, 1942, p.129: *Ma perchè hora in questo terzo intendo tractare de le degradationi de corpi compresi da diverse superficie et diversamente posti, pero avendo a tractare de corpi piu deficili, pigliaro altra via et altro modo nelle loro degradationi, che non o facto nelle dimostrationi passate; ma nello effecto sirà una cosa medessima e quello che fa l'uno fa l'altro. Ma per due cagioni mutaro l'ordine passato; l'uno è perchè sirà piu facile nel*

*dimostrare et nello intendere; l'altro si è per la gran multitudine de linee, che in essi corpi bisognaria de fare seguendo il modo primo, si che l'occhio et l'intellecto abagliaria in esse linee, senza le quali tali corpi non se possono in perfetione degradare, nè senza gran defilcultà. Pero pigliaro questo altro modo....*

[11] Francesco di Giorgio Martini, *Trattati*, ed. Corrado Maltese e Livia Maltese Degrassi, Milan: Edizioni il Polifilo,1967, vol. 1, f.33, Tav.61.

[12] For a more complete analysis see the author's *Leonardo da Vinci Studies I: Linear perspective and the visual dimensions of science and art*, Munich: Deutscvher Kunstverlag, 1986, pp. 68-86.

[13] G. B. Benedetti, *De rationibus operationibus perspectivae*, Turin: Apud Haeredem Nicolai Bevilaquae, 1585, p.119: " nescientes hunc solum esse verum modum". For an interpretation see the author's *Leonardo Studies I*, as in note 11, pp.392-395. See also J. V. Field, "Giovanni Battista Benedetti on the Mathematics of Linear Perspective", *Journal of the Warburg and Courtauld Institutes*, London, vol. 48, 1985, pp. 71-99.

[14] Pietro Accolti, Lo inganno degl'occhi, Florence: Appresso Pietro Cecconcelli, 1625, p.19:

*intende dimostrare delle due sudette operazioni, questa sola potere essere legittima , nel che stimiamo ingegnarsi sendo quella operazione la medesimea di questa.*

[15] Lionardo da Vinci, *Das Buch von der Malerei, III Band Commentar*, ed. Heinrich Ludwig, Vienna: Braumüller, 1888, p. 185.

[16] Erwin Panofsky, "Das perspektivische Verfahren Leone Battista Albertis", *Kunstchronik*, Leipzig, N.F., XXVI Jg., 1914-1915, Nr. 41-41, 6 August 1915, pp. 505-515.

[17] In the above mentioned article Panofsky refers to the distance point construction as *Distanzpunktverfahren*. Earlier authors sometimes referred to it simply as the distance point. See, for instance, Dr. H. C. O. Staigmüller, "Kannte Leone Battista Alberti den Distanzpunkt?", *Repertorium für Kunstwissenschaft*, Berlin, Bd. 14, 1891, pp. 301-304.

[18] For an illustration see the author's *Leonardo Studies I*, as in note 11, fig. 219.

[19] Alberti, as in note 2, p.38:

*Ma di questo diremone sue ragioni, se mai scriveremo di quelle dimostrazioni quale fate da noi, gli amici, veggendole e maravigliandosi, chiamavano miracoli.*

Cf. Ibid. p.42:

*Dicemmo de'triangoli, della pirramide, della intercesione quanto parea da dire; quale cose, mia usanza, soglio appresso de'miei amici prolisso con certe dimostrazioni ieometrice esplicare, quali in questi comentari per brevità mi parve da lassare.*

[20] Cf. Alessandro Gambuti, "Nuove ricerche sugli Elementa picturae", *Studi e documenti di architettura*, Florence: Teorema, n.1, 1972, pp. 133-172.

[21] Piero della Francesca, as in note 9, p.86: *Sia il piano del quale intendo recidere uno quadrilatero, che sia la sua lunghezza una quantità saputa, et la larghezza sia conoscuta; de la quale longhezza voglio tagliare la quantità de la larghezza che sia quadrilatera. Verbi gratia, sia il piano lungo 50 braccia, il quale sia poi degradato et sia BCDE, et BC sia 10 et BD sia 50. Prima che fusse degradato menaro la diagonale BE e perchè la larghezza che è 10 entra in 50, che è la lunghezza, 5 volte, pero faro de BC 5 parti, le quali seranno FGHI, e tiraro F al puncto A che deviderà BE diagonale in puncto K et tiraro K equidistante BC, che segarà BD in puncto L et CE in puncto M.*

*Dico aver reciso del pian degradato uno quadrilatero, il quale è BLCM et provase. Facciase uno quadrilatero in propia forma, il quale, sia largo 10 braccia et longho 50, il quale sia NOPQ, et NP sia 50 et NO sia 10 et menise la diagonale NQ. Poi devida[se] NO in cinque parti equali in RSTV et tirise R equidistante NP, la quale deviderà la diagonale in puncto X et deviderà PQ in puncto Y et mena X equidistante NO, che deviderà NP in puncto z et OQ in puncto Z, le quali faranno uno quadrilatero, che sirà NOzZ in propiia forma reciso dal piano NOPQ, condocto da la diagonale passante per X, devidente la linea Ry, la quale è la quinta parte de NO, commo dissi. BCDE è la quantità del piano NOPQ, benchè BCDE he degradato et la linea BC facta equale ad NO, et è devisa in cinque parti equali; et presone una de le quinte cioè BF e tiratola al puncto A devidente la diagonale in puncto K, et tirato la equidistante pasante per K., devidente BD in puncto L et CE in puncto M, sicommo ho preso de NO la quinta parte, cioè R et quella tirata equidistante NP, devidente la diagonale in puncto X, et poi tiratoX equidistante NO, che devide NP in puncto z etOQ in puncto Z et perchè la diagonale devide la superficie in propia forma nella quinta parte NOzZ, cosi devide la diagonale la superficie degradata, commo per la precedente s'è mostro nella quinta parte.*

[22] Ibid, p.87: *Ma se la quantità non fusse saputa de la lunghezza del dicto piano nè la larghezza, tiraro dal puncto A la linea equidistante BC, de la quantità che o posto il termine a l'ochio dato, et qui fermaro il puncto O et da quello menero OC che deviderà la linea BD in puncto L. Dico BL havere levato del piano BCDE degradato la quantità de BC, la quale è BL. Menise L equidistante BC, che segarà la diagonale BE in puncto K et CE in puncto M; dico BLCM essere quadrilatero tagliato del piano non quadrato BCDE, perchè la linea se parte da l'occhio O e termina in C e devide BD in puncto L, si che C se rapresenta a l'occhio levato piu che B la quantità de BL commo per la 12a fu provato.*

[23] Francesco di Giorgio Martini, as in note 11:
*Anco per altro modo essi piani da diminuire sono.*

[24] One of the earliest detailed references to Baldassare Peruzzi's "regola ordinaria" is found in: Giacomo Barozzi, il Vignola, *Le due regole della prospettiva pratica*, ed. Egnatio Danti, Rome: Zanetti, 1583, p.72. For a discussion of this see the author's *Renaissance Optics and Perspective. A Study in Problems of Size and Distance*, D.Phil, Warburg Institute, 1975, pp. 206-213.

[25] Cf. L. Brion-Guerry, *Jean Pélerin Viator. Sa place dans l'histoire de la perspective*, Paris: Société d'Edition les Belles Lettres, 1962, pp.74-115

[26] Jane Andrews Aiken, *Renaissance Perspective: Its Mathematical Source and Sanction*, PhD., Harvard Univerasity, 1986.

[27] This is the topic of an unpublished article by the author: "Perspective, Instruments and the Scope of Optics".