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Computers as an Historical Tool for Mathematics, Science and Art

Presented at the annual Leonardo meeting in Brescia in 1991.
Published as an Appendix to: *Leonardo's Method*, Brescia: Centro Ricerche Leonardiane, 1993.

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1. Introduction

The quantitative advantages of computers are obvious. They enable access to hitherto unthinkable amounts of knowledge. In terms of historical material this is of great significance because it will make the search for sources (*ad fontes*) available to individuals who do not have access to the very few remarkable libraries (e.g London, Paris, Vatican, Washington) where this level of research has traditionally taken place. In the foregoing text we have suggested that there is more to this quantitative dimension than sheer numbers of documents or size of databanks. Without such enormous amounts of hitherto scattered materials many questions can scarcely be addressed with any depth: how surveying instruments spread across Europe; what methods they employed; to what extent these methods were supplemented by textual descriptions; to what extent the local variations decreased as individuals became aware that methods developed in Urbino, Nürnberg, or Antwerp could be applied elsewhere; to what extent one can map changing relations between practice and theory? Related to this are questions of how materials can be presented in new ways: to what extent can one use combinations of lists, maps, and images to bring into focus relations among different aspects of knowledge which were hitherto invisible? All these are long term goals.

Meanwhile there are also short term possibilities: computers offer many new qualitative methods for the interpretation of historical material. Rather than attempting an abstract analysis of such methods, this paper focusses on some specific examples in the context of Renaissance art and science which are being explored under the auspices of the Perspective Institute at the McLuhan Centre (University of Toronto). An IBM compatible AT using AUTOCAD 10 in conjunction with D BASE III Plus is being used to make visible in a new way the genesis of perspectival methods from Alberti to Leonardo and demonstrate links with transformational geometry (*de ludo geometrico*), conic sections, principles of square and cube roots and other aspects of Renaissance mathematics, science and art. The chief characteristics of this approach can be described in terms of animation, alternative constructions; geometry and number as well as play, each of which will be considered in turn.

2. Animation

One of the greatest problems in the understanding of mathematical diagrams is that they represent the conclusion of a series of steps which, especially when the case is complex, provide very little clue about the steps taken in arriving at this conclusion. Traditionally this has been assumed to be the purpose of the accompanying text. One reads through a laborious proof, retraces the steps taken by the mathematician in question and thus arrives at his conclusions. This problem has been compounded by the fact that the heritage of Euclidean geometry has favoured abstract methods of presentation. Hence the diagrams frequently show three-dimensional spatial situations in two-dimensional terms. This heritage continued with the Renaissance artists and mathematicians who developed linear perspective. As a result those who discovered a new method of three dimensional representation (ironically) codified their findings in abstract two dimensional diagrams.

Using animation techniques available in AUTOCAD 10,[1] Eric Dobbs has shown how one can make these geometrical diagrams in the early treatises on perspective much more comprehensible by retracing step by step the various stages of a perspectival construction. This has the great advantage of making visible the process as well as a the end product. Thus far reconstructions have been made of principle constructions by Alberti, Antonio Averlino (Filarete), Piero della Francesca, Francesco di Giorgio Martini, Leonardo da Vinci and Egnatio Danti. For the purposes of this article three examples from the treatise of Piero della Francesca are given. The first (fig. 35) reconstructs the spatial context implicit in Piero's use of a perspectivally foreshortened pentagon using the geometrical method. The second example (fig.36) illustrates six steps in reconstructing the three dimensional spatial context underlying Piero's three view drawing which combines a ground plan, elevation and frontal view. The third example (figs 38-39) shows a two and three dimensional reconstruction of Piero's diagram accompanying Book I, proposition 23.

These animated reconstructions have other advantages. They permit a modern viewer to see three dimensional consequences of diagrams and thus resolve ambiguities which the Renaissance artists and mathematicians did not see because they drew their diagrams in only two dimensions. For instance, traditional debates concerning the legitimate construction (*costruzione legittima*) and the distance point construction are clarified if their principles are demonstrated in terms of three-dimensional situations.

3. Alternative Constructions

Alberti, who wrote the first treatise on perspective (1435) presents a special case. He appears to have avoided diagrams altogether and relied entirely on verbal descriptions. (In any case the earliest known manuscripts of his *De pictura* are without illustrations). Alberti's text has given rise to at least three different interpretations. By reconstructing these three different versions of the same text, it is possible to see and assess their relative merits.

Alternative constructions are sometimes useful even when the text and accompanying diagram are unambiguous. Piero della Francesca's *De prospectiva pingendi* offers a case in point. When Piero described the construction of a foreshortened octagon he relied on the presence of a static diagram and thus drew a series of lines the significance of which remained unclear until the diagram was nearly complete. Using a computer one can retrace these steps precisely. In addition, with the aid of coloured lines one can use a different sequence of steps in order to make visible Piero's method in a new way.

4. Geometry and Number

During the Renaissance there were frequently two alternative demonstrations for basic mathematical principles. For instance, in the case of square roots there was both a geometrical solution which illustrated the principle using a diagram and an arithmetical solution which dealt with the problem in terms of numbers. To a certain extent the two methods were interdependent. The development of geometrical methods made visible and measurable a set of relations which could then be summarized numerically. By the late sixteenth century this numerical arithmetic solution was gradually replaced by an algebraic solution, while the geometric solution was increasingly forgotten. Paradoxically, this advance hid the visual phase of the experience which had made possible the algebraic abstraction. As a result historians of mathematics have tended to write the history of their subject as a gradual liberation from visual demonstrations and an evolution towards abstraction. Computer animations allow one to move back and forth easily between geometrical, numerical and algebraic versions of the same principles and thus understand connections among these in a new way.

5. Play

As we look more closely at Renaissance mathematics, art and architecture we discern that underlying a seemingly overwhelming variety of forms, there are a surprisingly small number of basic geometrical shapes. Leonardo da Vinci's architectural studies offer a fascinating case in point. Most of his ground plans for churches involve circles, semi-circles, squares, rectangles and octagons. Indeed many of these begin with an octagon around which these other geometrical elements are added in some combination. Systematic drawing tools such as AUTOCAD enable us to catalogue these combinations and identify which subset of these were actually used by Leonardo. This same method can be applied to Leonardo's studies of the geometrical game (*de ludo geometrico*), which again involves a surprisingly small number of basic forms combined in various ways. Using a computer one can reconstruct these shapes, see their equivalences both visually in terms of geometry and mathematically in terms of computed areas. Moreover one can see which subset of potential combinations he actually studied. A computer thus permits more than a simple recreation of the activities of historical individuals: it enables us to picture their horizons and the limitations thereof.

6. Plans

Implicit in these reconstructions are new possibilities of entering into mental spaces of the past. In terms of mathematics, computers can show us correspondences between geometry and number and help us to understand historical trends in the changing interplay of geometry, arithmetic and algebra: i.e. the interplay of visualization and abstraction. In terms of science, if we study the texts and instruments available at a given period we can gain a reasonable picture of the parameters of methods open to individuals living at that time. In the case of surveying instruments we can explore links between more accurate instruments and growing levels of precision in topographical views and maps. In the case of telescopes it is possible to gain some idea of how much is visible with the use of various lenses. A co-ordination of recorded sightings with historical facts about the history of lenses will thus enable us to create an approximate map of how the limits of the visible universe changed with time.

A similar approach can be used in all the domains of science affected by the concept of quantification: we can explore the interplay between the parameters of instruments to measure and the horizons of measurement. And as we have noted earlier a census of instruments available for measurement will be an important means of exploring this phenomenon, for it is not just the act of measuring that is important. One person using a proportional compass is very different from over 100 publications about this instrument in six European languages within 50 years. This kind of census taking will offer us new insights into the cumulative dimensions of knowledge that set Europe apart from the rest of the world. It will also bring to light new dimensions of the interdependence of science and technology that are a characteristic of the west.

In terms of art it will be possible to catalogue themes and subjects treated as well as individual spatial and other elements and trace their history. The continuity of images will thus come into focus in a new way. If the evidence of painting practice is compared with that of the theoretical literature on art (for which von Schlosser provided an essential guide), then we shall be able to explore the changing interplay of practice and theory. In the context of some fields such as ornament and architecture where variations are to a certain extent predictable, it will become possible to map the extent to which individuals in the past have employed these forms in their designs of the man made world. In short we shall be able not only to reconstruct essential steps in the creativity of historical figures but also to gain some sense of ways in which the horizons of creativity have changed with time.

7. Conclusions

Every major breakthrough in civilization has involved a reorganization of knowledge. Greek civilization introduced not just the Academy but also the idea of storing knowledge in written form. Arabic civilization at Gundishapur in the eighth century began with a more systematic approach to collecting and translating the great texts known at the time. High mediaeval culture which saw the remarkable *Summa* of Saint Thomas Aquinas would not have been thinkable without access to a much greater corpus of knowledge by means of a great manuscript collection in Paris. In the Renaissance, the printing press enabled Leonardo to have 119 books and have access to many more through his wide circle of learned contacts. As we have tried to show, this access plus his systematic experiments with machines were the essential ingredients that made his new insights possible.

In our day computers are obviously marvelous new tools for storing historical facts in a more compact form. There is every reason to believe that they will also prove to be much more than this. The animation techniques they provide allow us to reconstruct earlier methods and to demonstrate alternative versions, such that we not only understand the value of these earlier explanations more clearly, but also learn why it was necessary to go beyond them. Computers are leading to both a reorganization and new interpretation of our cultural heritage. If there is some predictability of historical trends we are at the interstices of a new breakthrough in civilization. It might be appropriate if one of the insights of this new stage were a fresh understanding of the vision that inspired Leonardo five centuries ago.

Figures

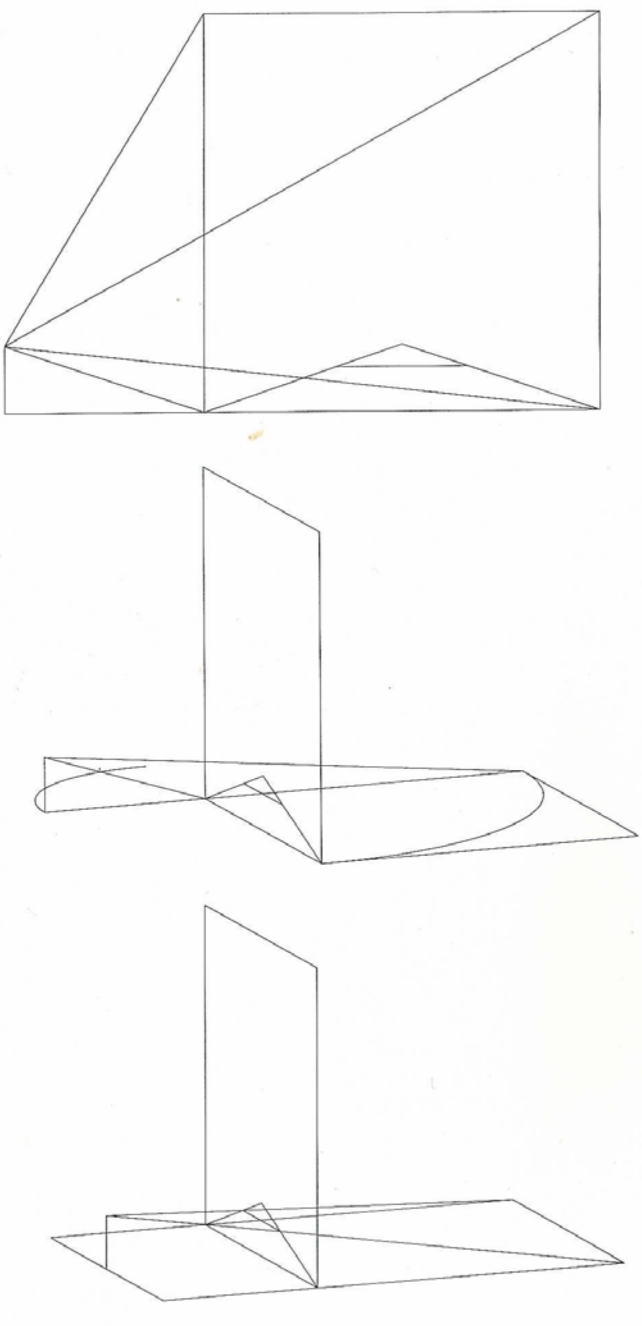
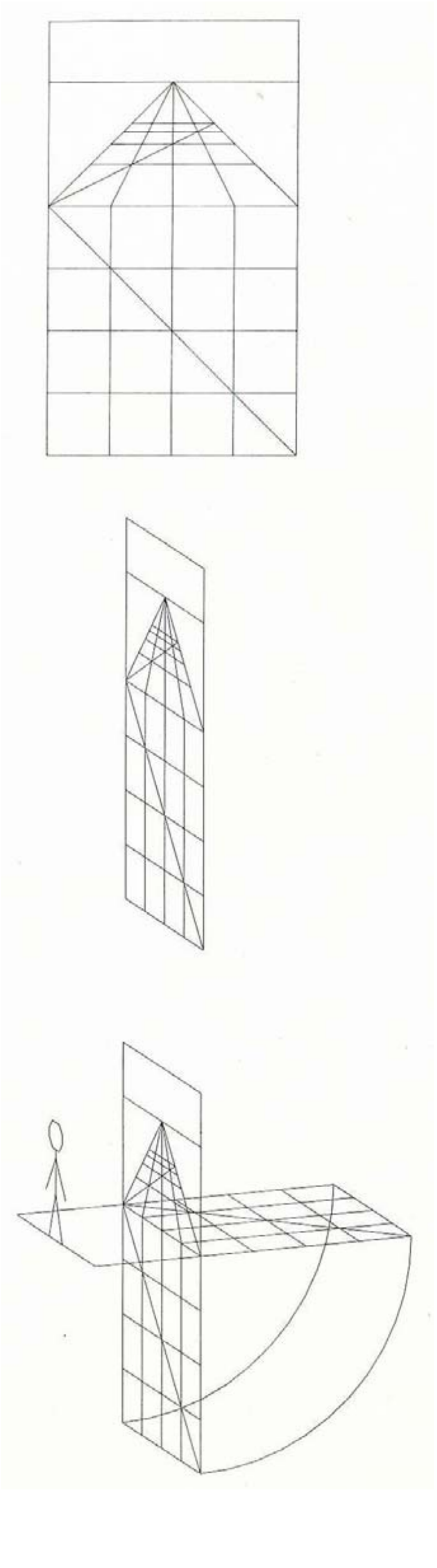
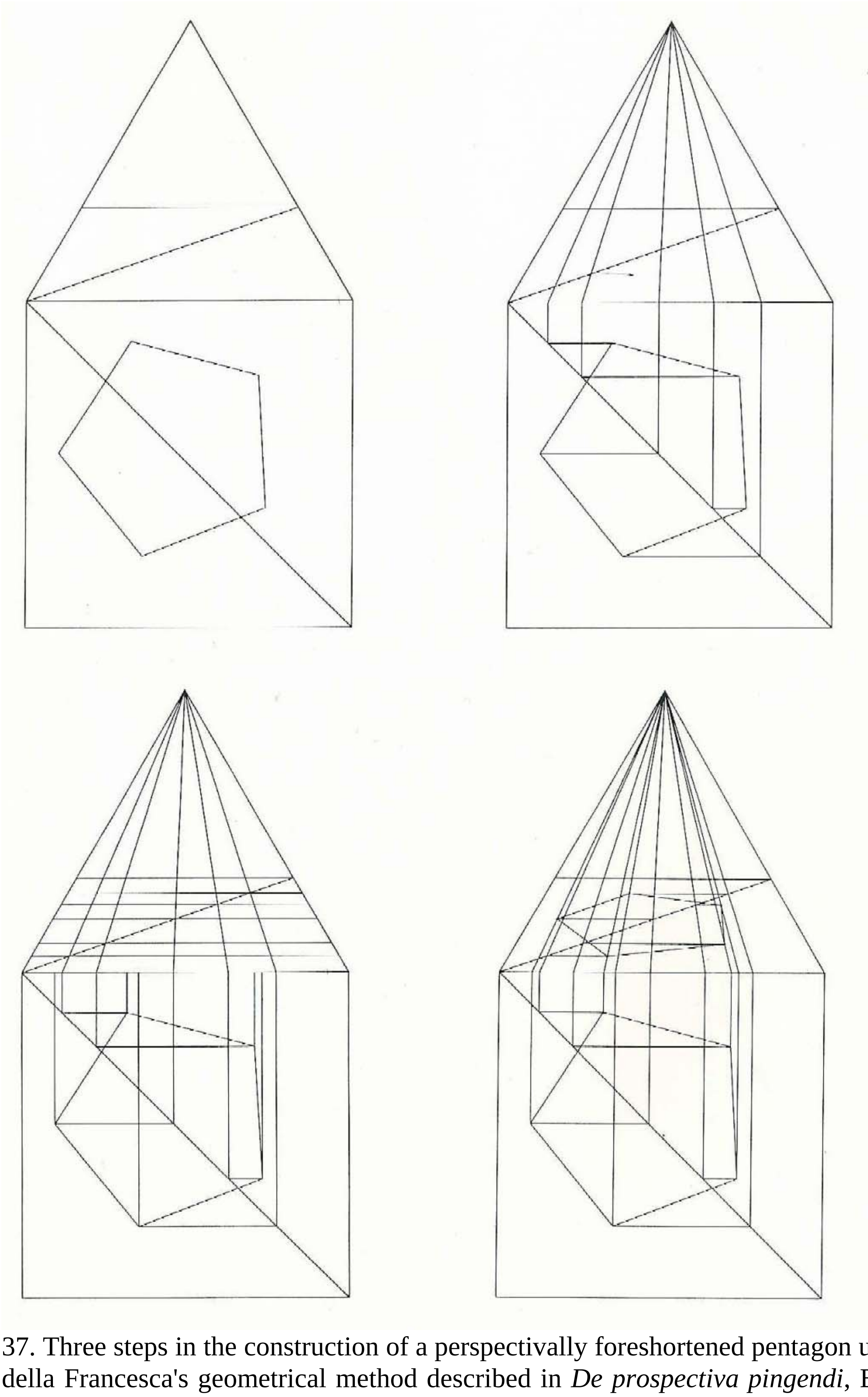


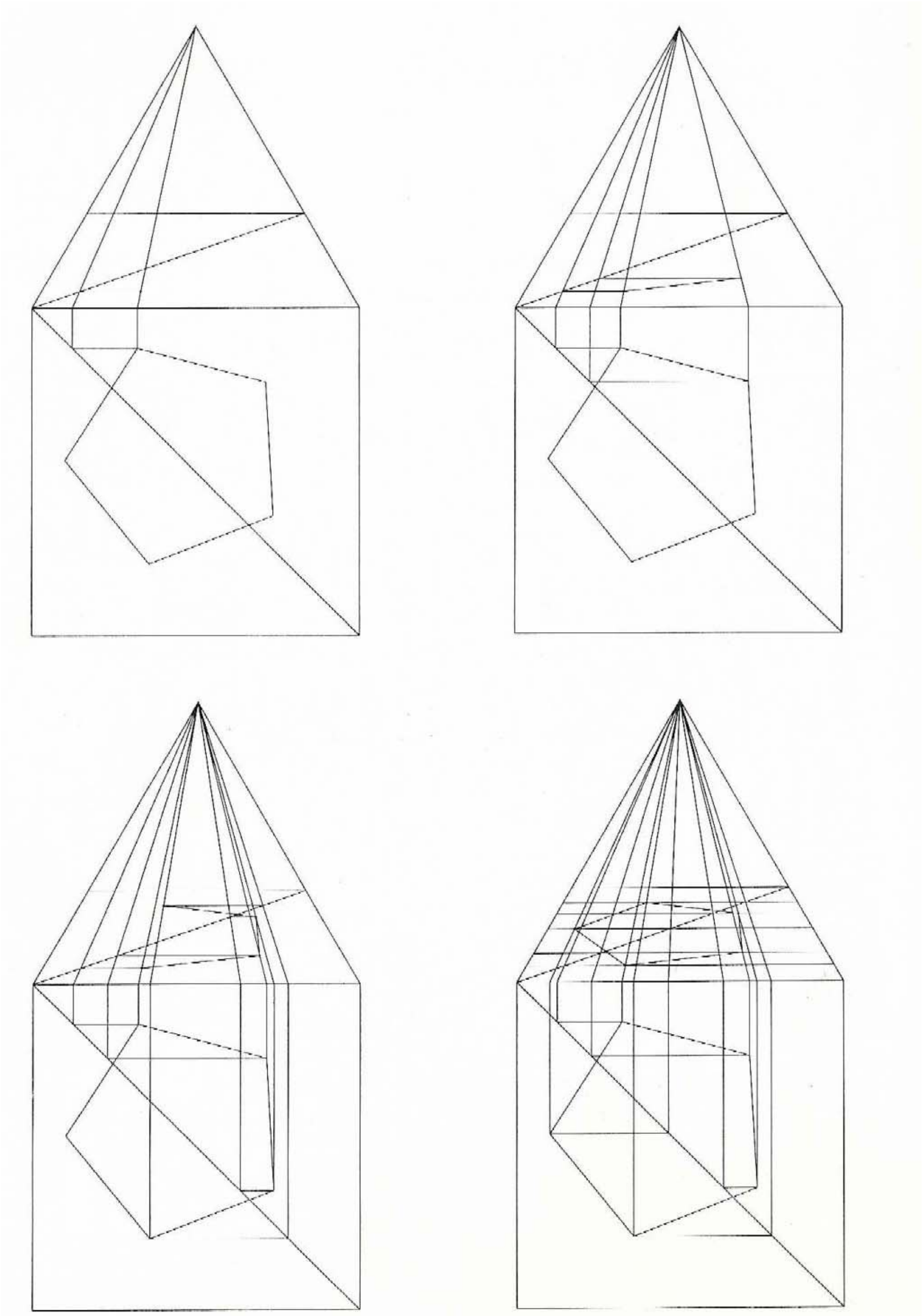
Figure 35. Piero della Francesca, *De Prospectiva pingendi*, Bk. 1, prop. 13 and two steps in the three dimensional reconstruction of same. The use of rotatiob renders animate version of these drawings much more effective.

Figure 36. Piero della Francesca, *De prospective pingendi*, Bk. 1, prop. 15, and two steps in three dimensional reconstruction of same.





37. Three steps in the construction of a perspectivally foreshortened pentagon using Piero della Francesca's geometrical method described in *De prospectiva pingendi*, Book 1.20. Diagrams by Eric R. Dobbs.



38. Four steps in the construction of a perspectivally foreshortened pentagon using Piero della Francesca's geometrical method described in *De prospectiva pingendi*, Book 1.20. Diagrams by Eric R. Dobbs.

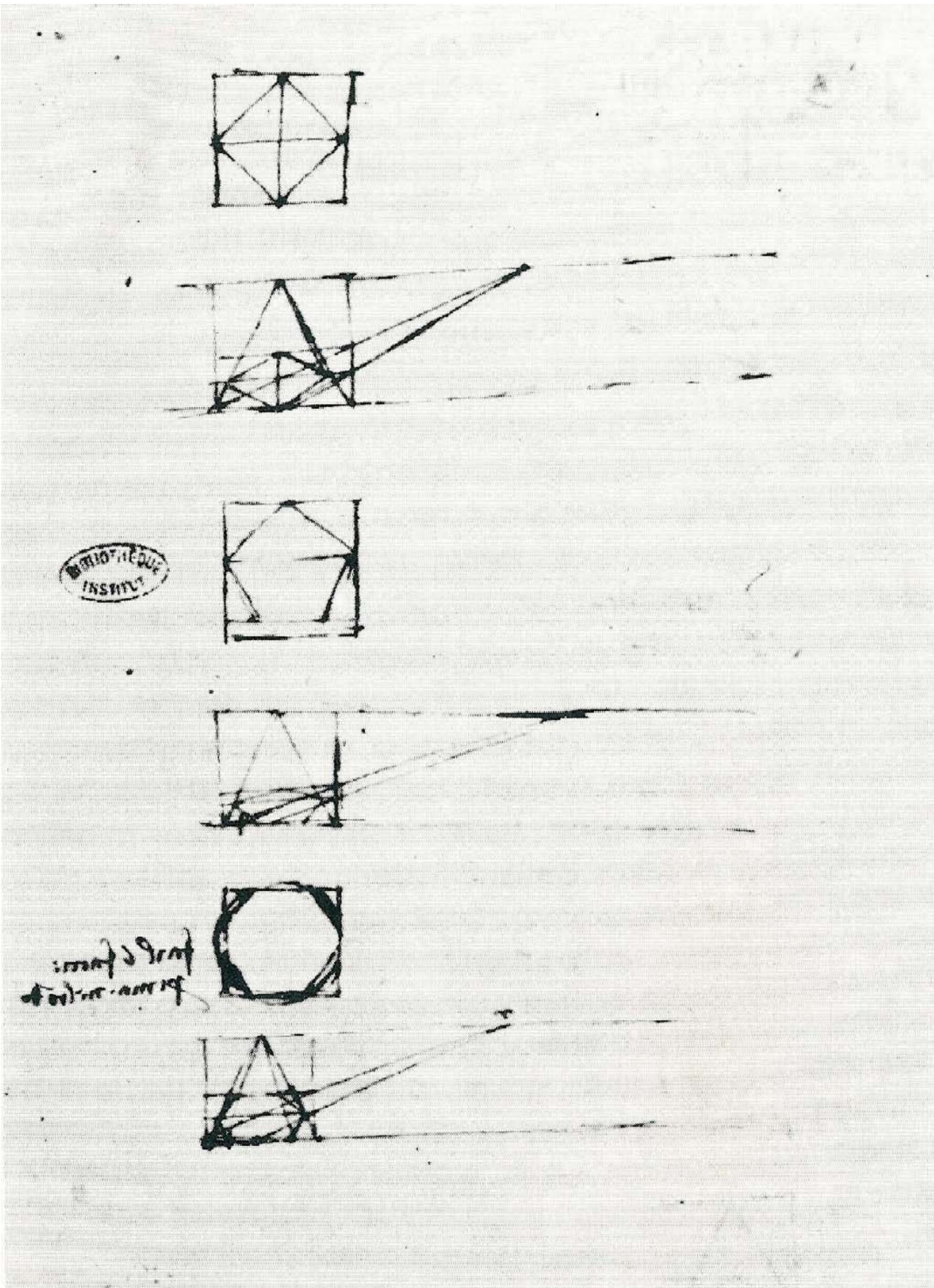


Figure 39. Leonardo da Vinci, Manuscript A, fol. 39r

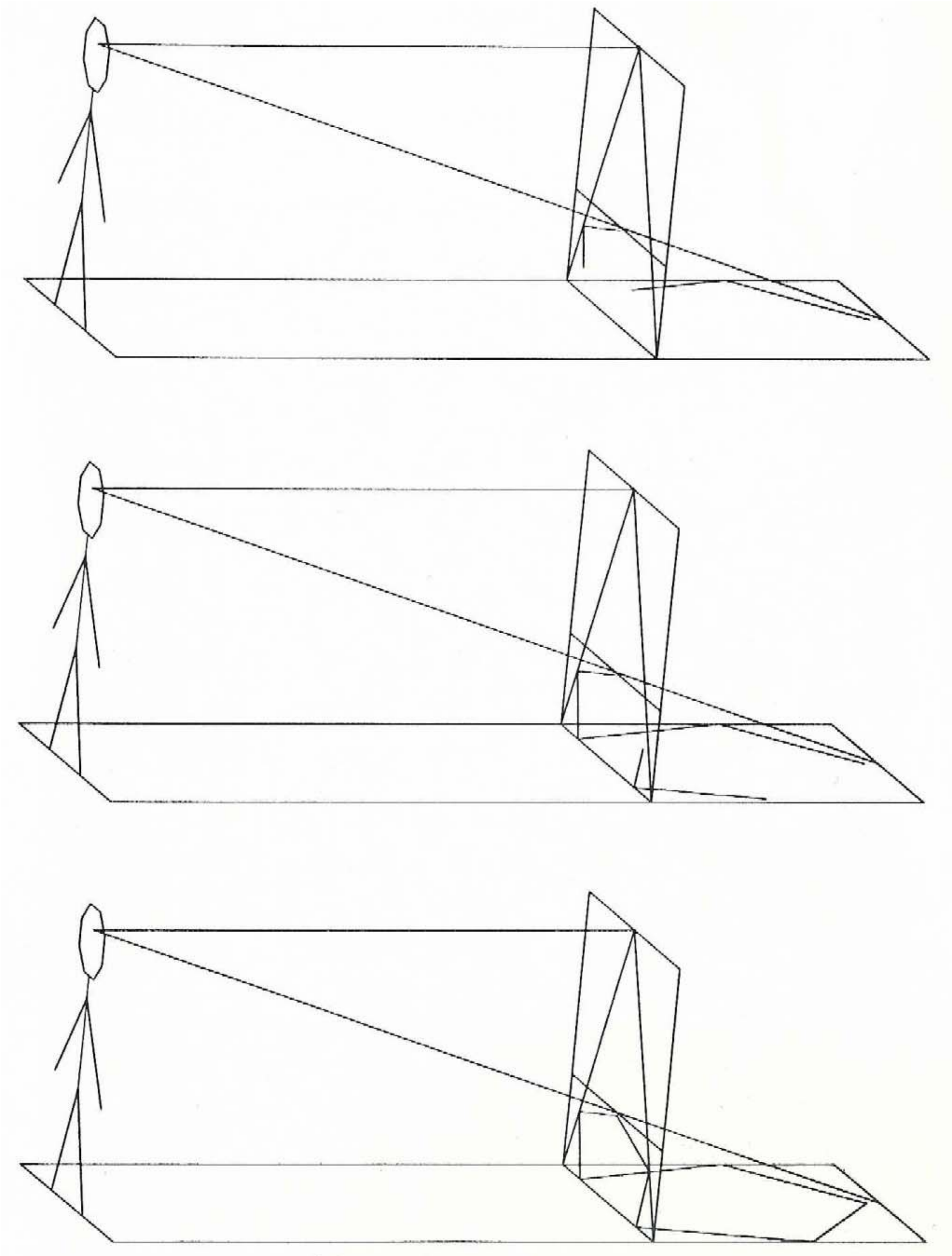


Figure 40. Three steps in the reconstruction of a pentagon illustrated on Manuscript A, fol. 39r.

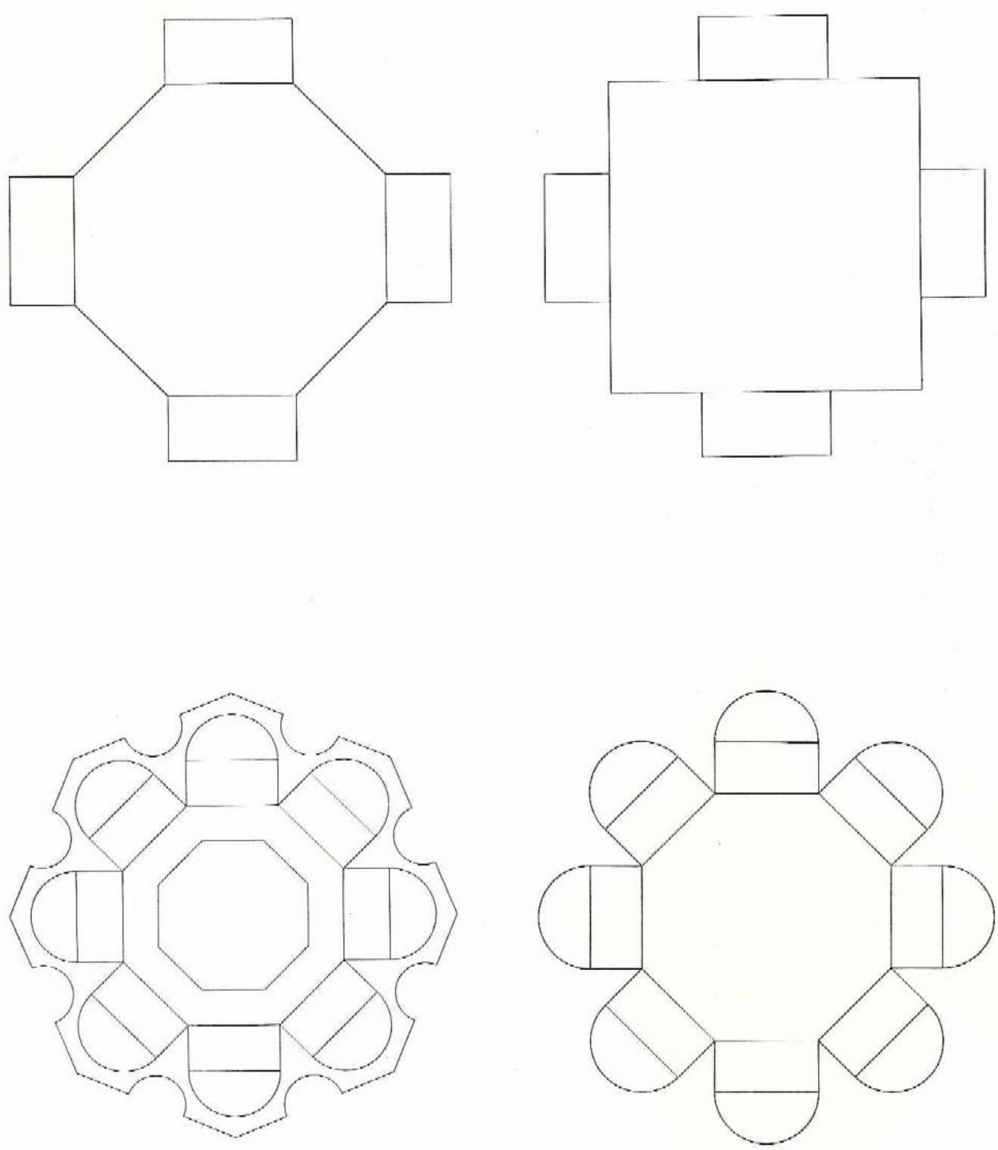


Figure 41. Four steps in growing complexity of geometrical shapes in architectural ground plans, Cf. above pl. 21.

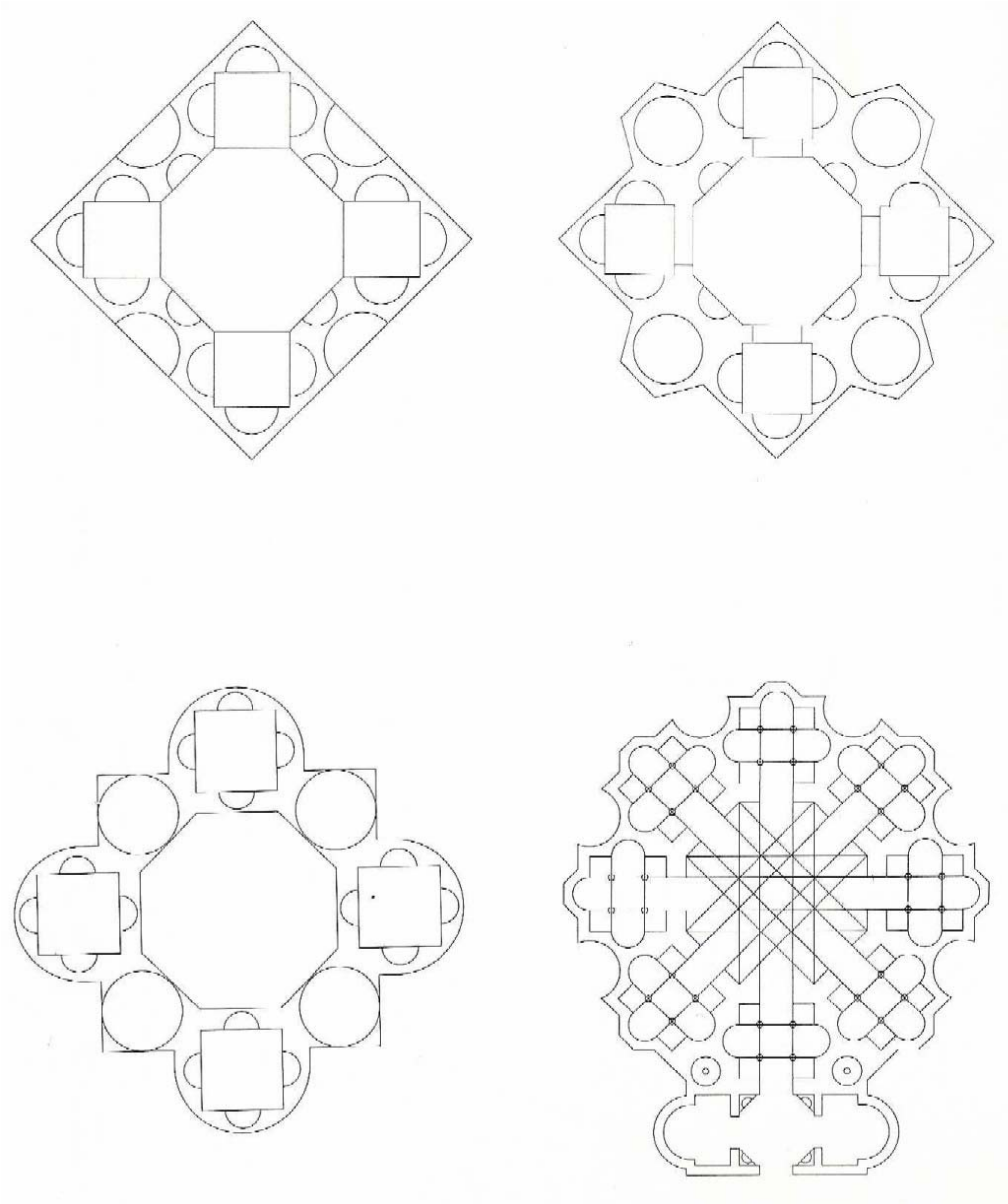


Figure 42. Four further steps in growing complexity of geometrical shapes in architectural ground plans, Cf. above pl. 21.

Notes

[1] The Perspective Institute at the McLuhan Centre is grateful to have been chosen as a test site for AUTODESK products.