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Piero della Francesca and the Two Methods of Renaissance Perspective

Published in: *Convegno internazionale: Piero della Francesca 500 anni, 1492-1992*, Urbino, Arezzo, Sansepolcro, October 1992, 1996, pp. 407-419.

1. Introduction

Our study of the past depends in no small part on the tools at our disposal. The latter half of the nineteenth century saw the emergence of a new attention to the theoretical sources of art. Critical in this regard was the monumental series *Source Books for Art History and Art Technique* (*Quellenschriften für Kunstgeschichte und Kunsttechnik*) edited by Eitelberger von Edelberg. This brought new editions of Alberti and Leonardo da Vinci as well as first editions of Filarete and Piero della Francesca. These editions led to new attention to details concerning the methods of Renaissance linear perspective. A series of mainly Austrian and German scholars: Ludwig, von Oettingen, Staigmüller and Janitschek drew attention to two different methods.[1]

Erwin Panofsky[2] established the standard view of these two chief methods of Renaissance perspective: the legitimate construction (*costruzione legittima*) and the distance point construction (*Distanzpunktverfahren*). The first, he claimed was a theoretical method that began in Italy, whereas the distance point construction was a practical method that began in the North and did not come to Italy until 1583 with Danti's edition of Vignola's *Two rules of practical perspective* (*Le due regole della prospettiva pratica*). Most scholars have simply accepted this view. Klein (1961)[3] linked the distance point construction with Northern workshop practice and suggested that there were connections with Pomponius Gauricus.

Brion Guerry (1962)[4] and Carter (1970)[5] questioned this interpretation, noting that since perspective was originally discovered in Italy it was likely that both methods would have originated there. Veltman (1975)[6] in an unpublished dissertation offered a first serious challenge to Panofsky's position. He agreed with Panofsky that the legitimate construction was discovered by Brunelleschi and codified by Alberti in his *On Painting*. But whereas Panofsky claimed this method was purely theoretical, Veltman claimed that it had evolved from a practical use of a window method. Moreover he claimed that there were two methods in use in Italy from the outset, that the second method had a theoretical basis in Euclidean geometry and was first described by Alberti in his *Elements of*

Painting. Veltman (1986)[7] returned to these claims in a brief appendix where he presented conclusive evidence for links between the legitimate construction and a practical window method.

Piero della Francesca is the earliest extant author who discussed explicitly two distinct methods. His statements on these methods and their significance are a concern of this paper. We shall suggest that Piero is building on distinctions already present at the time of Alberti. We shall then show that Piero's two methods are not identical to those described in Danti's edition of Vignola (1583). Hence the two methods, hitherto described as static conventions, were themselves changing throughout the Renaissance.

2. Piero's First Method

In *De prospectiva pingendi*, following a brief introduction, Piero outlines the organization of the three books:

> In the first we shall speak of points, lines and plane surfaces. In the second we shall speak of cubic bodies, of square pilasters of columns that are round and those with a number of faces. In the third we shall speak of heads and capitals, bases, torches of many bases and other bodies positioned in diverse ways.[8]

At the beginning of part three, Piero summarizes what he has done in the first two books and announces that he will use a different method in book three:

> In the first I demonstrated the diminution of plane surfaces in a number of ways. In the second I demonstrated the diminutions of square bodies and those having a number of faces positioned at right angles to the planes.
> But as I now intend in this third book to deal with the diminution of bodies which are more difficult, I shall describe another way and another means of diminishing them which I have not done in the foregoing demonstrations but in terms of effect it will be all the same thing if one uses one (method) or the other.[9]

Hence the use of two different methods is governed in part by the complexity of the object at hand as outlined in figure one.

Method One	Method Two
Books One, Two	Book Three
Simple Bodies	Complex Bodies
Points, Lines, Surfaces	Heads
Cubes, Square Pilasters	Capitals and Bases of Pillars

Figure. 1. Basic differences in objects treated using methods one and two.

In proposition six of book one Piero cites theorem 10 of Euclid's *Optics*. This is an exception. In the propositions that follow Piero repeatedly cites Euclid's *Elements*. When

we examine his procedure throughout books one and two, we find that it entails mainly geometrical proportion and proportional diminution. Propositions (figures) 9-11 involve unforeshortened shapes. Beginning with proposition 15 (figure 15a) through 22 Piero draws unforeshortened shapes in conjunction with their foreshortened equivalents. This continues in propositions (figures) 25-29.

It is significant that in the description cited above Piero mentions that he has demonstrated these things "in a number of ways". Propositions (figures) 12 and 13 demonstrate these principles in terms of surveying experience. Proposition 14 assumes this surveying context but uses an abbreviated demonstration. This surveying context is again implicit in proposition 23 which involves a diagonal method. Hence book one actually involves no less than three methods: one based on geometrical proportion using diagonals, another based on surveying using an interposed rod or plane (close to that which we now associate with the legitimate construction) and a third using a diagonal to determine the distance (related to that which we now associate with the distance point construction). Piero's categories are clearly very different from ours. Most of his propositions in the first two books are based on geometrical diminution. The examples from surveying and optics are used to confirm that optical experience coincides with geometrical demonstration.

Piero's first method can be traced back to Alberti's *Elements of Painting* (*Elementa picturae*) which contains similar diagrams relating to geometrical proportion. This proportional approach is also evidenced in Filarete's treatise.

2. Piero's Second Method

At the beginning of book three of his *On Perspective of Painting*, Piero explains why he is introducing a second method:

> But I shall change the arrangement used thus far for two reasons, one being that it will be easier to demonstrate and to understand. The other (reason) is on account of the multiplicity of lines which one would have to draw to these bodies if one followed the first method, with the result that the eye and the intellect will be deceived and without them these bodies cannot be perfectly diminished, nor without great difficulty. Hence I shall describe this other method by means of which I shall undertake to show the diminutions part by part. In this method, as I mentioned at the beginning of the first book, it is necessary to understand what a person wants to do and to know how to represent this properly on a plane for if they are placed in their propper form then the power of the lines by means of art will reproduce them diminished on the picture plane by the lines of sight. Thus it is necessary to know how to make all the contours commensurate with that which a person wants to make and place them on the plane in their places in their proper forms. I shall (now) outline this method in the demonstrations which follow.[10]

Hence Piero gives two reasons for introducing this second method. First, it is easier to demonstrate and to understand. Second, in terms of complex objects, it is less

cumbersome in terms of lines. Piero describes his method in detail which we shall again cite at length:

> Now to demonstrate the way that I intend to follow I shall give two or three examples of plane surfaces in order that by means of these you may more readily understand the diminution of bodies. So let there be constructed in proper form a square surface which is BCDE and then mark the point A which will be the eye and it will be as far back as you wish to stand to see this surface at the point A. Now fasten a nail or, if you will, a nail with a very fine silken thread, the hair of a horse's tail wuld be good, especially when it is far from the line. A line FG is then drawn parallel to BC which will be the picture plane between the eye and the surface . On this surface mark a point M which must be made on each surface and on each body. It makes no difference where you make it because it is a certain limit as you will understand as you go on. Now one will need a strip of wood that is very thin and straight. Then take one of these strips and lay it such that it is flush with FG and make sure that it is firmly positioned. Then take one end of the thread and take it to B of the place and where it touches the strip of wood make a point B. Then stretch the thread to C and where it touches the wooden strip draw M. Now mark an A on the wooden strip called the wooden strip A and this strip is then taken away and laid to the side. This is the wooden strip that indicates width. Now one needs to see how much higher DE of this plane BCDE is than BC. One therefore positions A as high above the line CE as one wants to stand to look at the plane in question, while neither moving towards or away from FG which marks the picture plane. After the eye has been fixed, as I described earlier, one takes a strip of paper and one places it contiguous to FG and draws EC which which meets the strip of paper at the point A, which will be the (paper) strip A. Then one takes the thread to E and at the palce where it touches the strip of paper one makes the points C and B at this same spot. One then takes away the strip of paper and one makes with it another that is identical to it with the same markings. And it will likewise be marked A as in the other one. Then draw a straight line in the place where you wish to make the perspectivally diminished plane, namelyt the line EG and divide it in two at point M and above M draw a perpendicular to I which will then become FH and GI. Then take the two strips of paper marked with A: one is placed contiguous with to GI and (the point) A of both lies on the line FG. Then one takes the strip of wood for width and one lays it over the two strips of paper such that one goes through E and D of the two strips (of paper) and M lies on the line MN and where the D of the the strip of wood touches the place make a point D and where E touches draw E. One now brings the wooden strip further down. such that it passes through B and C of the two strips of paper and M lies on the line MN and where B falls mark the point B and where C meets the strip of wood make the point C and the plane is drawn. Take away the strip of wood and draw BC,BD,DE,EC which is the diminished square plane which we said we would make.[11]

This detailed description is of great interest for several reasons. It confirms the Italian origins of a method that Dürer published in his *Instruction in Measurement* (1525), which

was subsequently published in Italy by Egnazio Danti in his *Two Rules of Practical Perspective* (1583). It confirms that this method which Panofsky termed the legitimate construction was a method based on practice rather than theory. It also confirms that there were two distinct methods being used in Italy during the fifteenth century. It will be useful if now reconsider what have been described as the two chief systems of the Renaissance.

3. The Geometrical Method

In his *On Painting*, Alberti[12] explicitly stated that there he was planning to write up geometrical proofs for perspective at a later date which he subsequently did in his *Elements of Painting*. The early manuscripts of the *Elements of Painting* have diagrams: the early manuscripts of *On Painting* do not. In Alberti's mind a second method, based on geometry, provided the proof and theoretical foundation for his practice. Hence there was no need for diagrams in *On Painting* which focussed on practice and at the same time was dedicated to readers with a rhetorical and literary context. After Alberti had written *Elements of Painting* this geometrical method became the main method in terms of theory. This helps explain why most of the diagrams in Filarete's treatise focus on this geometrical method.

Piero della Francesca continued this tradition. Hence he dedicated books one and two of his *On Perspective of Painting* to this geometrical method and book three to the method that has been remembered as the legitimate construction. Piero had a further reason for this arrangement. At the outset the window method was probably applied to simple geometrical shapes such as the eight sided Baptistery. But by the latter half of the fifteenth century, it was recognized that the geometrical method dealt more efficiently with such geometrical forms while the window was best applied in the context of more complex shapes. Implicit in this division was a basic distinction. The geometrical method served in creating general spatial contexts through pavements, ceilings and facades of buildings: the window method served in representing architectural and other individual details.

While Piero continued the tradition he also transformed its context. Unlike Alberti he was not content that geometry alone provided a viable demonstration that the principles of perspective were true. Hence he consciously prefaced his geometrical demonstrations with examples based on optics and surveying. Implicitly Piero was shifting the claims for proof from Euclidean geometry to a combination of geometry, optics and surveying. After giving these preliminary examples, Piero was content to rely on strictly geometrical principles in applying the method to triangles, pentagons, hexagons and other regular polygons, returning to an optical demonstration only once in the case of proposition 23.

Francesco di Giorgio followed Piero's example by giving surveying examples of both methods. Leonardo took this one step further by giving demonstrations for each of the regular polygons using the methods of optics and surveying. Optical demonstrations were now seen as more important than geometrical ones. Consequently the geometrical

method which had been the first method in Filarete and remained so in Piero and Francesco di Giorgio was now relegated to become the second method.

In Alberti's *On Painting* the diagonal functioned primarily as a test to confirm that the construction had been well done. In Alberti's *Elements of Painting* the diagonal became a basic part of the construction. In Piero's *On Perspective of Painting* (Book I, 23) the diagonal became linked with the distance point. In Francesco di Giorgio's demonstration a single diagonal implicitly became the starting point for a series of foreshortened shapes. According to Danti it was Baldassare Peruzzi who made this principle explicit terming it the ordinary rule (*regola ordinaria*). We know from Danti that Peruzzi's notes were inherited in large part by Serlio and we know also that Serlio was clearly aware of links between this geometrical method and the distance point even though he appears not to have understood its details.

Hence the geometrical method was associated with both the use of diagonals and the concept of a distance point in fifteenth century Italy. And while the documented evidence shows that it was through Pélerin in the North that these ideas were first developed in the way now associated with the so called distance point construction principle, it seems very likely that this would have evolved directly from the Italian circles deriving from Francesco di Giorgio Martini. In any case there can be no doubt that the geoemtrical method which began with a theoretical basis in Euclidean geometry was transformed sometime between 1490 and 1505 to become a method in which a distance point and surveying principles played a central role. These were its characteristics when it was codified by Danti as his second method. Hence it was that what began as a theoretical method could could subsequently be interpreted by scholars as a pratical one.

5. Legitimate Construction.

We have shown that what Panofsky described as the legitimate method was not derived from theory but was actually a practical technique involving the use of a window with its origins in a combination of optics and surveying. Alberti, in his *On Painting*, believed that the window or veil was essential to this method that he called the best way (*modo optimo*). It is frequently assumed that this so called legitimate construction depended on a combination of a ground plan and elevation. Vasari attributed the discovery of this method to both Brunelleschi and Uccello. However as Panofsky (1927) noted the first documented occurence of such a combination is found in Piero della Francesca. What is usually overlooked is that this occured in the first proposition of book three. Books one and two were devoted to what we have termed the geometrical method. Hence what is commonly termed the legitimate method was Piero's second method. It was not until Leonardo's practical demonstrations that this method moved into first place, a tendency confirmed by Serlio and codified by Danti in his commentary on Vignola's *Two Rules of Pratical Perspective* (1583). Hence far from being static both of the chief methods of Renaissance perspective underwent a series of changes in the century and a half from Alberti's first formulations to the Danti's codification of their principles. Figure two provides a summary of these developments.

Method based on optics, surveying (vision) window and thread (legitimate construction)	Method based on geometry (mathematics) proportion theory (distance point construction)
Brunelleschi?	
Alberti, *On Painting*, Bk.1	Alberti, *On Painting*, Bk.II Alberti, *Elements of Painting*
Filarete, *On Architecture* (method two)	Filarete, *On Architecture* (method one)
Piero, *On Perspective*... (method two)	Piero, *On Perspective*... (method one)
F. Di Giorgio Martini *Treatises* (method two)	F. Di Giorgio Martini *Treatises* (method one)
Leonardo da Vinci *Manuscript A* (method one)	Leonardo da Vinci *Manuscript A* (method two)
Dürer, *Instruction*	Dürer, *Instruction*
Serlio, *Architecture Bk. II* (method one)	Serlio, *Architecture Bk. II* (method two)
Danti, Vignola, *Two Rules* (method one)	Danti, Vignola, *Two Rules* (method two)

Figure 2. Summary of the two chief methods of perspective in the Renaissance

6. Conclusions

The second half of the nineteenth century which brought a new attention to theoretical sources of art history, led to a new interest in the technical methods of Renaissance perspective. Two methods were identified. They were treated as if they were entirely static. It was assumed moreover that both had effectively performed identical functions in

creating a single, homogeneous, isotropic concept of space. This was the picture which Panofsky codified in his *Pespective as Symbolic Form* (1927).

In retrospect it is clear that most of these assumptions were wrong. Yes, there were two principle methods during the Renaissance. But as we learn from Barbaro and Danti there were a number of competing methods not all of them accurate. Guidobaldo del Monte (1600) identified no less than 22 alternative methods.

The two chief methods both had Italian origins. The so called legitimate construction which Panofsky described as theorretically based had its roots in the window demonstration. The so called distance point construction appears to have had its origins in Italian geometrical theory rather than Northern workshop practice. Rather than remaining static, both methods enjoyed a dynamic development.

Both methods seem also to have had different functions. By the time Piero wrote his treatise the geometrical method was used primarily for creating geometrical contexts in terms of floors ceilings and regular architectural features, while the second method was used primarily in the representation of complex individual shapes, bases and capitals of columns, human heads and goblets. Ironically it was the geometrical method rather than the so called legitimate method that was the chief instrument of Renaissance contextual space.

In retrospect it can be seen that this fascination with contextual space was more closely linked with late nineteenth century neo-Kantian preoccupations with formal aspects of space than with the realities of fifteenth century art. If we look closely at Piero's cycle of the *True Cross* (Arezzo, San Francesco) we recognize that there are two distinct uses of perspective. One (the geometrical method)uses strings and vanishing points to establish the general spatial context. A second (the picture plane method the results of which were minutely applied using dots to copy from a cartoon onto the surface of the wall) was used in the case of the heads and other complex individual objects.

Interestingly enough the heritage of the nineteenth century has meant that histories of perspective have focussed almost entirely on the first of these methods. As a result paintings such as Piero's *Flagellation*, the three ideal cityscapes (Baltimore, Berlin and Urbino) and Leonardo's *Last Supper* have remained our archetypes of Renaissance perspective. The complex history of how artists mastered the spatial treatment of more complex shapes has yet to be told. Five hundred years after Piero's death we are only beginning to recognize the complexity of the new methods which he helped to articulate.

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Notes

[1] Bibliographical details concerning these are found Hermann Schüling, *Theorien der malerischen Linear-Perspektive vor 1601*, Giessen, 1973 and in Luigi Vagnetti, *De*

naturali et artificiali perspectiva, Firenze: Teorema, 1979 and will also be discussed in the author's upcoming bibliography.

[2] Erwin Panofsky, "Die Perspektive als symbolische Form", *Vorträge der Bibliothek Warburg*, Leipzig, 1927. This essay marked a development of ideas expressed as early as 1914 as was pointed out in the author's "Panofsky's Perspective: A Half Century Later", *La prospettiva rinascimentale. Codificazioni e trasgressioni*, a cura di Marisa Dalai Emiliani, Fireze: Centro Di, 1980, pp. 565-584.

[3] Robert Klein, "Pomponius Gauricus on Perspective, *Art Bulletin*, vol. 43, 1962, pp. 211-230.

[4] Liliane Brion Guerry, *Jean Pélerin, Viator*, Paris: Société d'Edition Les Belles Lettres, 1962.

[5] B.A.R. Carter, "Perspective", *The Oxford Companion to Art* edited H. Osborne, Oxford: Oxford University Press, 1970.

[6] *Renaissance Optics and Persapective. A Study in the Problems of Size and Distance*, D. Phil., Warburg Insitute, London University, 1975.

[7] *Linear Perspective and the Visual Dimensions of Science and Art*, Munich: Deutscher Kunstverlag, 1986 (Leonardo da Vinci Studies, 1).

[8] For the purposes of this discussion I shall rely on the edition by G. Nicco Fasola emendated by Eugenio Battisti of of Piero della Francesca, *De prospectiva pingendi*, Firenze: Casa editrice le lettere, 1984, p. 65:

Intese le sopradecte cose, seguitaremo l'opera, facendo di questa parte dicta prospectiva tre libri. Nel primo diremo de puncti, de linee et superficie piane. Nel secondo diremo de corpi chubi, de pilastri quadri, de colonne tonde et de piu facce. Nel terzo diremo de le teste et capitelli, base, torchi di piu base et altri corpi diversamente posti.

[9] Ibid, p. 129:

Nel primo dimostrai le digradationi de le superficie piane in piu modi;nel secondo ho dimostrato le degrationi de corpi quadri et de piu facce, posti perpendicularmente sopra de li piani.

Ma perche hora in questo terzo intendo tractare de le degradationi de corpi compresi da diverse superficiie et diversamente posti, pero avendo a tractare de corpi piu deficili, pigliaro altra via et altro modo nelle loro degradationi, che non o facto nelle dimostrationi passate; ma nello effecto sira una cosa medessima e quello che fa l'uno fa l'altro.

[10] Ibid, pp.129-130:

Ma per due cagioni mutaro l'ordine passato; l'uno e perche sira piu facile nel dimostrare et nello intendere; l'altro si e per la gran multitudine de linee, che in essi corpi bisogniaria de fare seguendo in modo primo, si che l'occhio et l'intellecto abagliaria in esse linee, senza le quali tali corpi non se possono in perfetione degradare, ne snza gran deficulta. Pero pigliaro questo altro modo, col quale porro parte per parte dimostrare le degradationi, nel qual modo, commo dissi nel principio del primo, e necesario intendere quello che l'omo vol fare et quello sapere ponere in propia forma, la forza de le linee seguendo l'arte le produranno degradate, sicommo se rapresentano nel termine dalle linee visuali. Perho e de bisogno sapere fare tucti li contorni mensuratamente de quello che l'omo vuol fare, et quello ponere sopra il piano nelli luoghi loro in propia forma, del qual modo daro notitia nelle dimostrationi che seguitarrano.

[11] Ibid, pp.130-131:

Hora per dimostrare il modo il quale intendo conseguire, faro do o tre dimostrationi de superficie piane, accio che per quelle posiate piu agevolmente venire a la notitia de la degradationi de'corpi. Adunqua, facise in propia forma una superficie quadrata, la quale sia BCDE; poi se punga il puncto A il quale sia l'occhio et sia tanto da lungi quanto si vuole stare a vedere la dicta superficie. Nel puncto A se ficchi il chiodo, o vuoli uno acho con uno filo di seta sutilissimo, siria buono uno pelo di coda una linea equidistante BC, la quale sia FG che sia il termine tra l'occhio e la superficie, nella quale superficie fa un puncto, che sia M, il quale se debbe fare in onni superficie et in onni corpo. No fa niente dove se faccia, perche e uno certo termine commo operando cognoscerai.

Hora bisogna avere righe de legno bene sutili et diricte; poi pi [glia u]na di queste righe, et polla contingente FG che stia bene ferma; poi piglise uno capo del filo di seta, et tirisesopra B de la superficie et dove bacte su la riga fa puncto B; poi se stenda il filo sopra C, e do'bacte su la riga seg[n]a C; poi se meni il filo sopra D, e dove bacte su la riga puncta D; tirise il filo sopra E, dove percote su la riga segna E; stendase il filo sopra M, e dove percote su la riga segna M. Fa hora una A su la riga, che se dica riga A, ew levise via, e pongase da canto, che e la riga de la larghezza.

Hora se vole vedere quanto e piu elelvato DE de questa superficie BCDE, che non e BC; adunque alzise A sopra la linea CE quanto se vole soprastare a vedere la dicta superficie, non s'acostando ne dilongando de la linea FG che e termine. Posto l'occhio A col filo, commo dissi, facciase una riga di carta et pongase contingente FG et menise EC devidente le riga de carta in puncto A che sira riga A; poi se tiri il filo sopra E, e dove bacte su la riga de carte segnase E et D; poi se stenda il filo sopra C, e dove bacte su la riga se faccia puncto C et B in uno medesimo luogo; poi se tolga via la rigfa, et con quella se ne faccia un'altra similecon quelli medessimi segni, et sia segnata A como l'altra.

Da poi mena la linea recta nello luogo dove tu voi fare la superficie degradata, la quale linea sia FG, et devidila per equali in puncto M et sopra M tira la perpendiculare che sia MN et tira sopra F, H perpendiculare , et sopra G tira I perpendiculare , che siranno FH et GI; poi piglia le do rige de carta segnata A: una se ponga contingente FH, et l'altra continga GI et A de tucte do continga la linea FG. Poi se pigli la riga de legno segnata A, che e la riga de la larghezza, et pongase sopra le do riga de carta, contingente E et D de tucte do le rige et M continga la linea MN et dove combascia D de la riga de legno fa puncto D, et dove combascia E segna E; tirise la riga contingente B et C de le do rige et M continga la linea MN, et dove combascia B punta B et dove combascia C de la riga da legno fa C; et e fornita la superficie.

Leva via le rige et tira BC,BD,DE,EC, che fia la superficie quadrata degradata che dicemmo fare.

[12] Brunelleschi's role in this development remains uncertain. The past century has devoted considerable energies to reminding us that the six lines of Manetti's second hand account of Brunelleschi's discovery were imprecise. It is generally accepted that sometime between 1415-1425 Brunelleschi made his famous drawing of the Baptistery while standing in the doors of Santa Maria del Fiore. It is very likely that a transparent surface

(velo, probably cloth, possibly glass) was one of the key elements in producing this image just as a mirror was basic for its viewing. It is striking that Alberti calims that he invented the velo in the Latin version of On painting but omits this claim in the Italian version dedicated to Brunelleschi.